COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OF PHASE CONSTANT FOR DIPOLE MODE SURFACE WAVE PROPAGATION IN OPEN GYROMAGNETIC FERRITE ROD

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SUMMARY

The formulation of Maxwell's equations as an eigenvalue problem in operator notation [1] enables application of operator methods, Schroedinger scalar perturbation theory, and mode orthogonality conditions to the solution of propagation problems in uniform waveguides. Gabriel and Brodwin [2] initiated this approach for obtaining approximate solutions for waveguiding problems involving inhomogeneous, anisotropic, dissipative media in conventional waveguide in which the presence of the media was considered a perturbation of the conditions present in the empty waveguide. Lee [3] extended this theory to open gyrotropic dielectric waveguide by considering the gyrotropy of the rod to be a perturbation of the conditions present in an open isotropic dielectric waveguide. In particular, Lee obtained a perturbation solution for waveguiding of the HE_{11} (dipole) surface wave mode in a longitudinally magnetized open gyromagnetic ferrite rod by considering the anisotropy of the ferrite induced with the application of the small dc magnetic field.

Considering first order perturbations only, the rod phase constant (eigenvalue), β , may be expressed [3]:

$$\beta = \beta^{(0)} + \beta^{(1)} \tag{1}$$

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NOTES

where: $\beta^{(0)}$ ≡ zero order phase constant for dielectric rod of same relative dielectric constant and dimensions as ferrite rod

$\beta^{(1)}$ ≡ first order perturbation term which includes the anisotropic effects of the ferrite

For isotropic rods of relative permeability, μ_r, and relative dielectric constant, ϵ_r, $\beta^{(0)}$ is determined from [3, 4, 5]:

$$\frac{\beta^{(0)}}{k_o} = \left[\frac{P^2 + \mu_r \epsilon_r Q^2}{P^2 + Q^2}\right]^{\frac{1}{2}} \tag{2}$$

where: P ≡ radial eigennumber for fields inside the rod

Q ≡ radial eigennumber for fields outside the rod

$k_o = 2\pi/\lambda_o$

λ_o ≡ free space wavelength,

and P and Q are the solutions of the simultaneous equations [3, 4, 5]:

$$\frac{2\pi a}{\lambda_o} = \left[\frac{P^2 + Q^2}{\epsilon_r - 1}\right]^{\frac{1}{2}} \tag{3}$$

$$[\epsilon_r \eta_1(1, P) + \eta_2(1, Q)][\eta_1(1, P) + \eta_2(1, Q)] - \left[\frac{1}{P^2} + \frac{1}{Q^2}\right]^2 \left[\frac{P^2 + \epsilon_r Q^2}{P^2 + Q^2}\right] = 0 \tag{4}$$

and:

$$\eta_1(1, P) = \frac{J_1'(P)}{P J_1(P)} \qquad \eta_2(1, Q) = \frac{K_1'(Q)}{Q K_1(Q)}$$

a ≡ radius of rod

Equation (2) is plotted versus rod diameter to free space wavelength ratio for dielectric rods of various ϵ_r in Figure 1.

Lee's first order perturbation expression may be rewritten in the following form [5]:

$$\frac{\beta^{(1)}}{k_o X_1} = \frac{[\Delta_1^2 - \epsilon_r^2][C3][U]}{[2(C1)(\Delta_1^2 + \epsilon_r U^2) - 4(\Delta_1)(C2)(U^2 + \epsilon_r)]} \tag{5}$$

where: $\Delta_1 = \dfrac{[\epsilon_r \eta_1(1, P) + \eta_2(1, Q)]}{\left[\dfrac{1}{P^2} + \dfrac{1}{Q^2}\right]}$

$$C1 = \frac{1}{2}[P^2 J_0^2(P) + (P^2 - 2) J_1^2(P)]$$

$$C2 = \frac{1}{2} J_1^2(P) \qquad U = \frac{\beta^{(0)}}{k_0}$$

$$C3 = \frac{1}{2}[(P^2 + 2) J_0^2(P) + P^2 J_1^2(P) - 2]$$

Equation (5) is plotted versus rod diameter to free space wavelength ratio in Figure 2, and, in modified form, in Figure 3. X_1 is given by [6]:

$$X_1 = \frac{|\gamma^2| 4\pi M_s H_{INT}}{|\gamma^2|[H_{INT}]^2 - \omega^2} \tag{6}$$

where: γ ≡ gyromagnetic ratio

$4\pi M_S$ ≡ saturation magnetization of ferrite

ω = 2π (frequency)

H_{INT} ≡ internal dc magnetic field

The internal dc magnetic field for a longitudinally magnetized circular ferrite rod with dimensions small compared to a wavelength may be approximated by [7]:

$$H_{INT} = H_{APP} + 2\pi M_z + \frac{2|K_1|}{M_S} \tag{7}$$

where: H_{APP} ≡ applied longitudinal dc magnetic field

$4\pi M_z$ ≡ magnatization of ferrite rod along rod axis for a given applied dc magnetic field

K_1 ≡ first order anisotropy constant of ferrite

Figure 4 is a comparison of computed versus measured values of (dipole mode) rod wavelength to free space wavelength ratio as a function of longitudinally applied dc magnetic field for TT 1-390 ferrite of rod diameter to free space wavelength ratio of 0.1947 at 12.36 GHz for:

$$\frac{\lambda_g}{\lambda_0} = \frac{k_0}{\beta} \tag{8}$$

where: λ_g ≡ guide wavelength

The measured data was derived from experimental results obtained by Kott [8]. It was evident from the data that the ferrite had not been demagnetized for the measurements, and the calculated value for zero applied dc magnetic field was therefore computed for:

$$H_{INT}(0) = 2\pi M_r + \frac{2|K_1|}{M_S} \tag{9}$$

where: $4\pi M_r$ ≡ retentivity of ferrite

There is generally good agreement between predicted and measured results below a longitudinally applied dc magnetic field of 200 oersteds. (Typical applied dc magnetic fields for devices utilizing this mode of propagation are less than 50 oersteds [9].) The slight shape departure of the calculated curve from the measured curve at very low applied dc magnetic fields may be caused by the fact that the expression for X_1 given in equation (6) was derived for a saturated ferrite, a condition which does not obtain at extremely low applied dc magnetic fields.

Figure 5 shows the dipole mode rod wavelength to free space wavelength ratio as a function of longitudinally applied dc magnetic field for some commercial ferrite rods of rod diameter to free space wavelength ratio of 0.1947 at 12.36 GHz.

ACKNOWLEDGMENT

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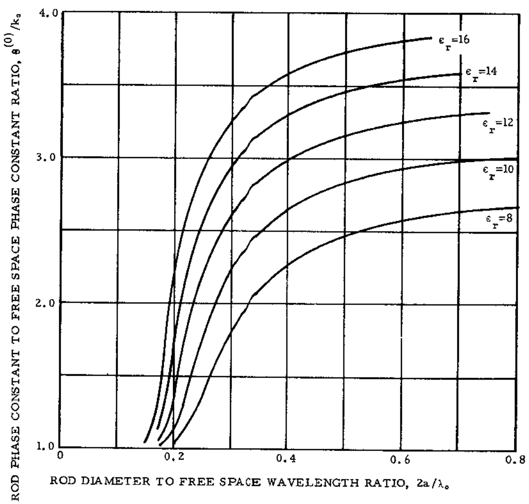

Figure 1 Dipole Mode Rod Phase Constant to Free Space Phase Constant Ratio as a Function of Rod Diameter to Free Space Wavelength Ratio for Isotropic Dielectric Rod of Relative Dielectric Constant, ϵ_r

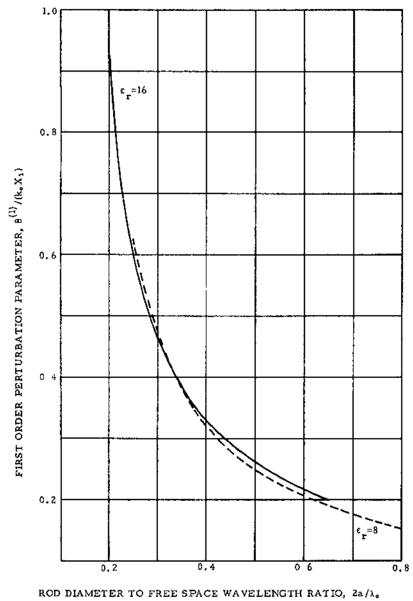

Figure 2 Dipole Mode First Order Perturbation Parameter as a Function of Rod Diameter to Free Space Wavelength Ratio for Ferrite Rod of Relative Dielectric Constant, ϵ_r

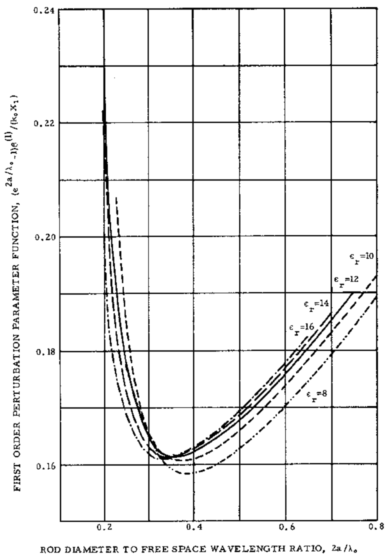

Figure 3 Dipole Mode First Order Perturbation Parameter Function as a Function of Rod Diameter to Free Space Wavelength Ratio for Ferrite Rod of Relative Dielectric Constant, ϵ_r

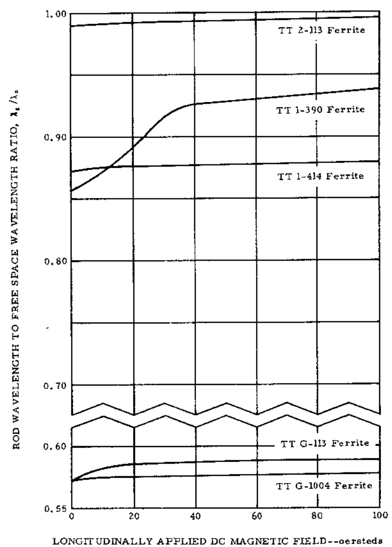

Figure 5 Dipole Mode Rod Wavelength to Free Space Wavelength Ratio as a Function of Longitudinally Applied dc Magnetic Field for Some Commercial Ferrite Rods of $2a/\lambda_o$=0.1947 at 12.36 GHz

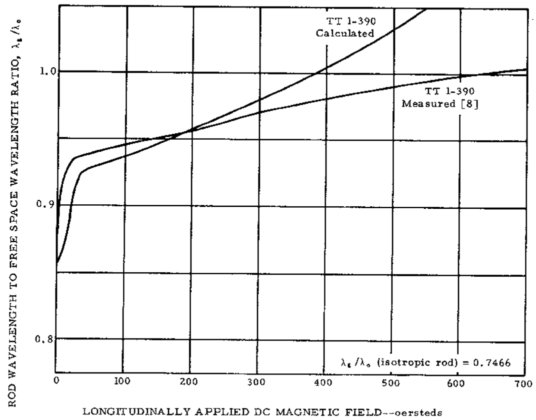

Figure 4 Theoretical and Measured Values of Dipole Mode Rod Wavelength to Free Space Wavelength Ratio as a Function of Longitudinally Applied dc Magnetic Field for TT 1-390 Ferrite Rod of $2a/\lambda_o = 0.1947$ at 12.36 GHz